Network Statistics in JAS

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1 Introduction

The paper explains the functioning of statistics available using the objects *NetworkStatComputer* and *VertexStatComputer*, contained in the *jas.graph.statistics* package of the JAS Library (see [JAS]).

Those objects can be instantiated by the model developer: the *NetworkStatComputer* is thought to be created once, computing statistics on the whole network, while a particular instance of *VertexStatComputer* has to be created for each node to be studied. The network object doesn't need instances of vertex objects to work.

The network object is constructed passing the *Graph* object[1] containing all network objects, while vertex objects are constructed passing them the vertex object they refer to, in the form of an object implementing the interface *IRelationalAgent* of the package *jas.graph*.

Statistics of both objects are accessible through the call of two methods, *getIntegerValue(int key)* and *getDoubleValue(int key)* which return integer and double values representing statistics asked via the key integer value. Those key values and data types (integer or double) are illustrated below for each object (see tables 1 and 6).

Since the statistics objects have been designed to be integrated in JAS and for considering dynamic networks of autonomous, hetereogenous and interacting agents (i.e. Agent Based Models), some choices have been made to adapt standard analyses coming from graph and network theories. The first choice has been to consider just the usage of directed edges in networks: considering the realization of ABMs about social realities, we consider that identical and bidirectional connections between actors are the exception and not the rule. Thus, the user must

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[1]The *Graph* object and the others representing vertices and edges belong to the JGraphT library (see [JGraphT]), contained in JAS.

take care of directions and weights of relationships, being able to organize homogeneous "undirected" edges as well as heterogenous ones. The presence in this package of generic and specific statistics lets the user choose the most effective tool: when a truly directed network is built, the "in" and "out" versions of statistics are more informative, but when the network is built upon symmetric edges, the mean computation made by "general" statistics is simpler.

Secondly each statistics comes from a formula established in the literature, but adapted to our framework[2]. The paper focuses on the computing process implemented, showing simple examples and helping the user to better understand and use such tools.

As a final remark, we suggest the reading of [Scott'91] for an introduction to social network analysis techniques, and the reading of [Wasserman&Faust'94] for a deeper survey of network statistics and literature.

2 Network Level Statistics

2.1 Explanation

The **number of vertices** is just the whole number of vertices in the graph, however they are connected. For the networks presented as examples, its value is 5. It is commonly represented by the letter N.

The **number of connected vertices** evaluates nodes belonging to the graph and incident at least to one edge. It doesn't mean that each connected node can reach each other connected node: it is possible that separate clusters are present, or that some nodes are unreachable because of directionality. Such data can be the same as the total number of vertices when each vertex is connected, as in figure 2, or different such as for the case of the graph of figure 3. It is represented by N_C.

The **inclusiveness** is the ratio between the number of connected vertices and the their total amount: $I = \frac{N_C}{N}$.

The **sum of degrees** is the sum of the degree value of all vertices. It can be also conceived as the double of the number of edges because each edge connects two vertices. It is very important to stress that using directed edges means to consider two edges for a bidirectional connections between two nodes. Thus, if node V1 points to node V2 and the opposite, there are two directed and parallel edges betweene them. In this case the degree of each node is 2, and the sum of degrees is thus 4 even if just one bidirectional connection is present. The sum is

[2]A particular adaptation of formulas is that, in literature, unreachable vertices are considered with a distance of ∞, while in our implementation those distances are simply ignored.

Statistics	**Key**	**Type**
Number of vertices	1	Integer
Number of connected vertices	2	Integer
Inclusiveness	3	Double
Sum of degrees	4	Integer
Sum of weighted degrees	5	Double
Average degree	6	Double
Average weighted degree	7	Double
Degree variance	8	Double
Weighted degree variance	9	Double
Average betweenness	10	Double
Betweenness variance	11	Double
Average eccentricity	12	Double
Eccentricity variance	13	Double
Number of edges	14	Integer
Sum of weighted edges	15	Double
Density	16	Double
Weighted density	17	Double
Diameter	18	Integer
Weighted diameter	19	Double
Degree based centralization	20	Double
Indegree based centralization	21	Double
Outdegree based centralization	22	Double
Betweenness based centralization	23	Double
Closeness based centralization	24	Double

Table 1: Network Level Statistics

computed by the following formula:

$$S = \sum_{i=1}^{N} d_i$$

The **sum of weighted degrees** is the sum of the weighted degree value of all vertices. It can be also conceived as the double of the number of edges multiplied for their weight. Even here it is important to stress the role of directed edges as above. It is computed by the following formula:

$$S_w = \sum_{i=1}^{N} d_{i,w} = \sum_{i=1}^{N}(\sum_{j=1}^{E} w_{ij} d_{ij}),$$

where $d_{i,w}$ is the weighted degree of vertex i.

The **average degree** is the mean value of degree among nodes. It is computed as follows: $\overline{d} = \frac{S}{N}$, i.e. as the ratio between the sum of degrees and the total number of vertices, connected or not.

The **average weighted degree** is similar to the previous one and is the ratio between the sum of weighted degrees and the number of edges: $\overline{d}_w = \frac{S_w}{N}$.

The **degree variance** is $S_d^2 = \frac{\sum_{i=1}^{N}(d_i - \overline{d})^2}{N}$, while the **weighted degree variance** is $S_{d_w}^2 = \frac{\sum_{i=1}^{N}(d_{i,w} - \overline{d}_w)^2}{N}$.

The **average betweenness** is $\overline{b}$ the mean value of all vertices betweenness values (see the following paragraph for a detailed description of this statistics), while the **betweenness variance** is $S_b^2 = \frac{\sum_{i=1}^{N}(b_i - \overline{b})^2}{N}$.

The same kind of computation is behind the **average eccentricity** ($\overline{e}$) and the **eccentricity variance** ($S_e^2 = \frac{\sum_{i=1}^{N}(e_i - \overline{e})^2}{N}$).

The **number of edges** computes the number of edges in the graph. As said before, it is important to take care of the presence of directed edges. This value is represented by the letter E.

The **sum of weighted edges** is the number of edges weighted for their weight and it is obtained as half the sum of weighted degrees: $E_w = \frac{S_w}{2}$.

The **density** is computed as the ratio between the number of edges present and the maximum amount of edges feasible for the graph. In our case the maximum amount of edges is $N(N-1)$ and not $\frac{N(N-1)}{2}$ as in common network statistics because we are considering directed edges. Thus the statistics is computed following the formula: $D = \frac{E}{N(N-1)}$.

The **weighted density value** is computed as a measure of the density and the intensity of edges in the network. It is the ratio between the weighted sum of edges and the maximum amount of edges (that is to say the amount $N(N-1)$ considering for each edge the weight of 1): $D_w = \frac{E_w}{N(N-1)}$.

The **diameter** of the graph is the greatest distance between any pair of its nodes. It is implemented considering the shortest path between any pair of vertices and getting the maximum value found. The shortest path search algorithm used is the Dijkstra's one. Even in this case it is important to underline the importance of using directed edges: the shortest path from V1 to V2 can be completely different from the one from V2 to V1. The algorithm implemented verifies each pair, swapping the source and the target. Formally, the statistics computed is the value $\Delta = \max_i \max_j \delta^{\rightarrow}(i,j)$ with $i, j \in [1, N]$ and $\delta^{\rightarrow}(i,j)$ representing the directed geodesic shortest distance (represented by the number of edges of the path) between i and j.

The **weighted diameter** is as the diameter but considering weighted shortest paths. The formula is $\Delta_w = \max_i \max_j \delta_w^{\rightarrow}(i,j)$ with $i, j \in [1, N]$ and $\delta(i,j)_w$ representing not the amount of edges in the path but their weights.

Passing to measures of network centralization, the first statistics available is the **degree based centralization** that evaluates how the network depends on one vertex. It is computed adpating the general centralization index, that lies between 0 and 1 (0 when all vertices have the same centralization index and 1 when one vertex completely dominates the network) and considers a general evaluator ($C_A(n_i)$) of each node centrality:

$$C_A = \frac{\sum_{i=1}^{N-1}[C_A(n^*) - C_A(n_i)]}{\max \sum_{i=1}^{N-1}[C_A(n^*) - C_A(n_i)]}$$

In this case we consider d, the degree value as the evaluator of network centrality and thus the numerator of the index is just the sum of differences between the highest degree value in the network and the each other degree value. On the contrary, the denominator is considered as $2(N-1)(N-2)$ that is equivalent to the difference between the maximum degree and the nodes degree value in a star network, the most centralized network. In fact, in such network, the maximum degree is the one of the star center, and it is $2(N-1)$, with the 2 representing the fact that each connection can be in two ways (because the graph is directed) and that $N-1$ nodes are connected to the center. Morevoer, the degree of other nodes is 2, thus $\sum_{i=1}^{N-1}[2(N-1)-2] = 2(N-1)(N-2)$ and the statistics formula becomes:

$$C_d = \frac{\sum_{i=1}^{N-1}[d(n^*) - d(n_i)]}{2(N-1)(N-2)}.$$

Evaluating the **indegree based centralization** of the network we consider, as measures of nodes centrality, the indegree value. The formula is similar to the previous one, but the denominator is half the previous one because we consider just inedges:

$$C_{d^{in}} = \frac{\sum_{i=1}^{N-1}[d^{in}(n^*) - d^{in}(n_i)]}{(N-1)(N-2)}.$$

The **outdegree based centralization** on the opposite considers the outdegree values of nodes. The formula adopted is:

$$C_{d^{out}} = \frac{\sum_{i=1}^{N-1}[d^{out}(n^*) - d^{out}(n_i)]}{(N-1)(N-2)}.$$

The last statistics available about network centralization is the one considering the betweenness values of nodes. The **betweenness based centralization** is thus computed considering those values on the numerator and the value $(N-1)$ on the denominator. In fact the greteast difference among betweenness values exists in a full centralized network, where the betweenness value of the central node is 1 and in all the $N-1$ other is 0. The formula thus becomes:

$$C_b = \frac{\sum_{i=1}^{N-1}[b(n^*) - b(n_i)]}{(N-1)}.$$

The **closeness based centralization** considers as basic statistics the vertex closeness, also called gloabl centrality (as in the following paragraph). The ratio denominator is, even in this case, the maximum possible, i.e. the one of differences in a star network ($\frac{(N-2)(N-1)}{2N-3}$). The whole formula is:

$$C_c = \frac{\sum_{i=1}^{N-1}[c(n^*) - c(n_i)]}{\frac{(N-2)(N-1)}{2N-3}},$$

in the case in which $N = N_C$, otherwise $C_c = 0$.

2.2 Examples

No. of vertices:	5	Eccentricity variance:	0.0
No. of connected vertices:	5	No. of edges:	20
Inclusiveness:	1.0	Sum weighted edges:	20.0
Sum of degrees :	40	Density:	1.0
Sum of weighted degrees:	40.0	Weighted density:	1.0
Average degree:	8.0	Diameter:	1
Average weighted degree:	8.0	Weighted diameter:	1.0
Degree variance:	0.0	Degree based centralization:	0.0
Weighted degree variance:	0.0	Indegree based centralization:	0.0
Average betweenness:	0.0	Outdegree based centralization:	0.0
Betweenness variance:	0.0	Betweenness based centralization:	0.0
Average eccentricity:	1.0	Closeness based centralization:	0.0

Table 2: Statistics of the Connected Network of Figure 1

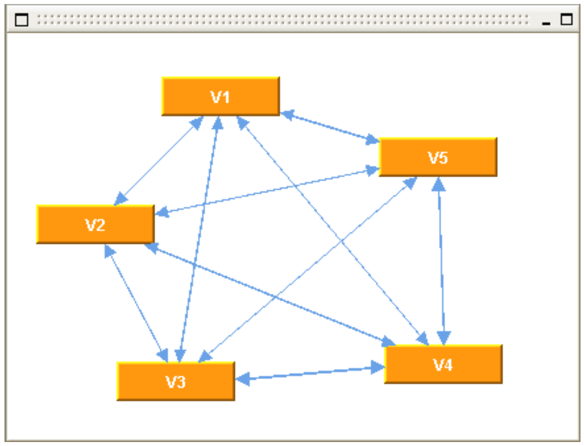

Figure 1: Fully Connected Network with 5 Vertices.

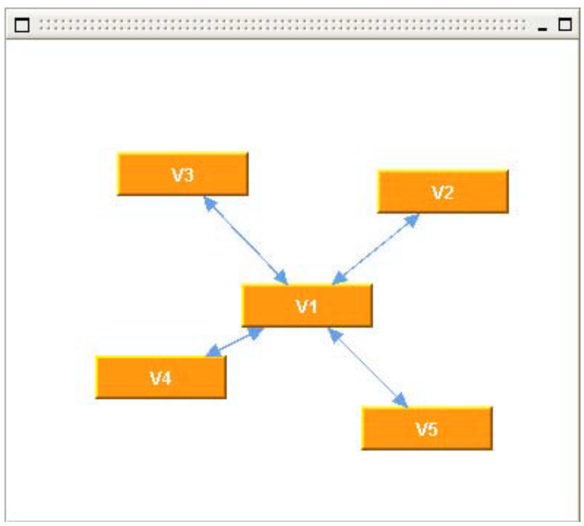

Figure 2: Star Network with 5 Vertices.

No. of vertices:	5	Eccentricity variance:	0.16
No. of connected vertices:	5	No. of edges:	8
Inclusiveness:	1.0	Sum weighted edges:	8.0
Sum of degrees :	16	Density:	0.0
Sum of weighted degrees:	16.0	Weighted density:	0.4
Average degree:	3.0	Diameter:	2
Average weighted degree:	3.2	Weighted diameter:	2.0
Degree variance:	5.8	Degree based centralization:	1.0
Weighted degree variance:	5.8	Indegree based centralization:	1.0
Average betweenness:	0.2	Outdegree based centralization:	1.0
Betweenness variance:	0.16000000000000003	Betweenness based centralization:	1.0
Average eccentricity:	1.8	Closeness based centralization:	1.0000000000000002

Table 3: Statistics of the Star Network of Figure 2

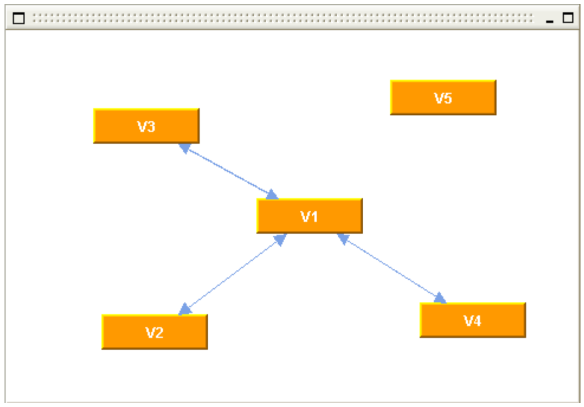

Figure 3: Star Network with 4 Vertices Connected and 1 Alone.

No. of vertices:	5	Eccentricity variance:	0.64
No. of connected vertices:	4	No. of edges:	6
Inclusiveness:	0.0	Sum weighted edges:	6.0
Sum of degrees :	12	Density:	0.0
Sum of weighted degrees:	12.0	Weighted density:	0.3
Average degree:	2.0	Diameter:	2
Average weighted degree:	2.4	Weighted diameter:	2.0
Degree variance:	4.0	Degree based centralization:	0.75
Weighted degree variance:	4.0	Indegree based centralization:	0.75
Average betweenness:	0.1	Outdegree based centralization:	0.75
Betweenness variance:	0.040000000000000015	Betweenness based centralization:	0.5
Average eccentricity:	1.4	Closeness based centralization:	0.0

Table 4: Statistics of the Star Network of Figure 3

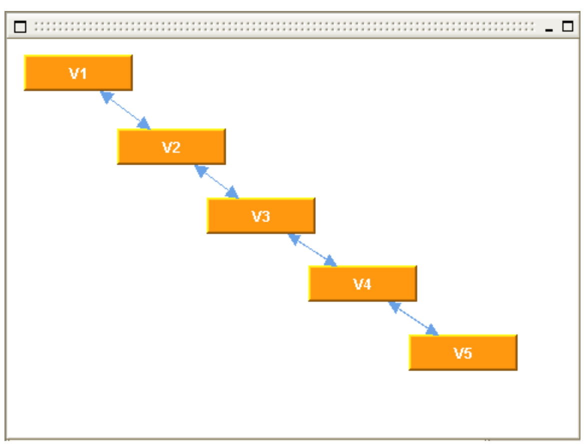

Figure 4: Linear Network with 5 Vertices.

No. of vertices:	5	Eccentricity variance:	0.5599999999999999
No. of connected vertices:	5	No. of edges:	8
Inclusiveness:	1.0	Sum weighted edges:	8.0
Sum of degrees :	16	Density:	0.0
Sum of weighted degrees:	16.0	Weighted density:	0.4
Average degree:	3.0	Diameter:	4
Average weighted degree:	3.2	Weighted diameter:	4.0
Degree variance:	1.0	Degree based centralization:	0.16666666666666666
Weighted degree variance:	1.0	Indegree based centralization:	0.16666666666666666
Average betweenness:	0.3333333333333333	Outdegree based centralization:	0.16666666666666666
Betweenness variance:	0.07777777777777778	Betweenness based centralization:	0.41666666666666663
Average eccentricity:	3.2	Closeness based centralization:	0.42222222222222217

Table 5: Statistics of the Linear Network of Figure 4

3 Node Level Statistics

3.1 Explanation

Statistics related to single vertices focus on the possible role that a vertex play in its surrounding and the whole network. For that reason, first statistics are concentrated on evaluating the vertex loacl centrality, while the last ones are on its global centrality.

The first measure of a vertex local centrality is its **absolute degree** value, i.e. half the sum of all its incidents edges: $d = \frac{E_v}{2}$, where E_v is the edge set of the vertex v considered. Thus, even in this case, it is important to underline that using direct edges, both connections starting from and arriving to the vertex are counted by this statistics, while the **absolute indegree and outdegree** consider just, respectively, the incident inedges (d^{in}) and outedges (d^{out}):

A way to better compare local centrality data for different vertices, even in different networks or surroundings, is to consider a relative value and not an absolute one. Thus, the **relative degree** value is the ratio between the absolute one and the maximum degree possible in the network, that is the fact of being connected to all other vertices through in and outedges: $d_{rel} = \frac{d}{2(N-1)}$.

The **relative indegree** instead considers just inedges ($d_{rel}^{in} = \frac{d^{in}}{(N-1)}$), while the **relative outdegree** just outedges ($d_{rel}^{out} = \frac{d^{out}}{(N-1)}$).

Considering the presence of different weights among edges, the **weighted degree** of a vertex is the sum of each incident edge, evaluated for its weight:

$$d_w = \frac{\sum_{e=1}^{E_v} w_e}{2}.$$

The **weighted indegree** statistics and the **weighted outdegree** one are similar to the one just mentioned, but considering weighted inedges and outedges:

$$d_w^{in} = \sum_{e=1}^{E_v^{in}} w_e,$$

Statistics	**Key**	**Type**
Absolute degree	1	Double
Absolute indegree	2	Integer
Absolute outdegree	3	Integer
Relative degree	4	Double
Relative indegree	5	Double
Relative outdegree	6	Double
Weighted degree	7	Double
Weighted indegree	8	Double
Weighted outdegree	9	Double
Absolute degree -2	10	Double
Absolute indegree -2	11	Integer
Absolute outdegree -2	12	Integer
Global centrality	13	Double
Global in-centrality	14	Double
Global out-centrality	15	Double
Weighted global centrality	16	Double
Weighted global in-centrality	17	Double
Weighted global out-centrality	18	Double
Corrected global centrality	19	Double
Corrected global in-centrality	20	Double
Corrected global out-centrality	21	Double
Weighted & corrected global centrality	22	Double
Weighted & corrected global in-centrality	23	Double
Weighted & corrected global out-centrality	24	Double
Betweenness	25	Double
Eccentricity	26	Integer
In-eccentricity	27	Integer
Out-eccentricity	28	Integer

Table 6: Vertex Level Statistics

$$d_w^{out} = \sum_{e=1}^{E_v^{out}} w_e.$$

While the standard degree is a measure of how many nodes can be reached in one step from the starting vertex, it could be interesting to evaluate not just the quantity of nodes reached but also the quality. For instance, a node connected to only one other could be in a good position for local centrality because the close one is very well connected. To measure such a second level centrality, the **absolute degree -2** can be used: it evaluates how many nodes can be reached in less than 3 steps, thus enumerating the vertex close neigbors and their close neighbors. The formula, using the Dijkstra shortest path algorithm to evaluate geodesic distances (δ), is as follows:

$$d_{-2} = \frac{\sum_{i=1}^{N-1} \sigma(v,i) + \sum_{i=1}^{N-1} \sigma(i,v)}{2},$$

with $\sigma(i,j) = 1$ if $\delta^{\rightarrow}(i,j) < 3$, 0 otherwise.

The **absolute indegree -2** uses the same concept but considers just inedges, arriving to the vertex considered. In other words, it enumerates the vertex that can be reached in two steps starting from there (v):

$$d_{-2}^{in} = \sum_{i=1}^{N-1} \sigma(v,i).$$

The **absolute outdegree -2**, on the opposite, evaluates how many vertices can reach the vertex considered in two steps:

$$d_{-2}^{in} = \sum_{i=1}^{N-1} \sigma(i,v).$$

The statistics called **global centrality** evaluates the role of the vertex on the whole network, considering the sum of shortest paths to reach other nodes as a measure vertex centrality. It is a relative measure, compared to the the centrality of a center in a star network ($N-1$):

$$g_v = \frac{N-1}{\frac{\sum_{i=1}^{N-1} \delta^{\rightarrow}(v,i) + \sum_{i=1}^{N-1} \delta^{\rightarrow}(i,v)}{2}}.$$

The **global in-centrality** evaluates the sum of shortest paths reaching the vertex, $g_v^{in} = \frac{N-1}{\sum_{i=1}^{N-1} \delta^{\rightarrow}(i,v)}$, while the **global out-centrality** is the sum of shortest paths leaving the vertex, $g_v^{out} = \frac{N-1}{\sum_{i=1}^{N-1} \delta^{\rightarrow}(v,i)}$.

The **weighted global centrality** is the the same as global centrality, but considering the edges composing shortest paths not as units but by their weights and thus considering an absolute measure and not a relative one:

$$g_{v,w} = \frac{\sum_{i=1}^{N-1} \delta_{w}^{\rightarrow}(v,i) + \sum_{i=1}^{N-1} \delta_{w}^{\rightarrow}(i,v)}{2}.$$

The **weighted global in-centrality** considers weighted shortest paths pointing to the vertex, $g_{v,w}^{in} = \sum_{i=1}^{N-1} \delta_{w}^{\rightarrow}(i,v)$, while the **weighted global out-centrality** is the weighted sum of shortest paths leaving the vertex, $g_{v,w}^{out} = \sum_{i=1}^{N-1} \delta_{w}^{\rightarrow}(v,i)$.

Another way to evaluate global centrality is to evaluate the importance of being connected to other vertices depending on their degree and on their distance, mixing information of local and global centrality. The **corrected global centrality** in fact is the sum of ratio between the vertices degrees and the length of the shortest path to reach them, a value then inverted and multiplied for $2(N-1)$, that is the maximum value for a star centralized network:

$$c_v = \frac{2(N-1)}{\frac{\sum_{i=1}^{N-1}(\frac{d_i}{\delta^{\rightarrow}(v,i)} + \frac{d_i}{\delta^{\rightarrow}(i,v)})}{2}},$$

where the global centrality of vertex v depends on distances with each other vertex and their degree (d_i).

The **corrected global in-centrality** considers how much other nodes take to reach the vertex considered and their outdegree: $c_v^{in} = \frac{N-1}{\sum_{i=1}^{N-1} \frac{d_i^{out}}{\delta^{\rightarrow}(i,v)}}$, while the **corrected global out-centrality** considers how much it takes to reach other nodes and their indegree: $c_v^{out} = \frac{N-1}{\sum_{i=1}^{N-1} \frac{d_i^{in}}{\delta^{\rightarrow}(v,i)}}$.

Considering the possibility to set different weights in edges, the **weighted and corrected global centrality** evaluates the vertex global centrality by computing the sum of the weighted paths with other nodes, weighted also by their weighted degrees:

$$c_{v,w} = \frac{\sum_{i=1}^{N-1}(\frac{d_i,w}{\delta_{w}^{\rightarrow}(v,i)} + \frac{d_{i,w}}{\delta_{w}^{\rightarrow}(i,v)})}{2},$$

where $d_{i,w}$ is the weighted degree of vertex i and $\delta_{w}^{\rightarrow}(v,i)$ is the weighted directed shortest path between v and i.

The **weighted and corrected global in-centrality** uses the connections arriving at the vertex and the weighted outdegrees of other vertices, $c_{v,w}^{in} = \sum_{i=1}^{N-1}(\frac{d_i^{out},w}{\delta_{w}^{\rightarrow}(i,v)}$, while the **weighted and corrected global out-centrality** considers weighted paths starting from the vertex and indegrees of other vertices: $c_{v,w}^{out} = \sum_{i=1}^{N-1}(\frac{d_i^{in},w}{\delta_{w}^{\rightarrow}(v,i)}$.

The **beetweeness** value is, as considered in literature, an evaluation of the role of the vertex as an hub for connections among other vertices. It is computed evaluating the ratio between the numebr of times the vertex belongs to shortest paths connecting each pair of other nodes possible and the maximum possible (that is $(N-1)(N-2)$):

$$b_v = \frac{\sum_{i=1}^{N-1} \sum_{j=1}^{N-1} \delta^{\rightarrow}(i,j)}{(N-1)(N-2)}.$$

The vertex **eccentricity**, called also the vertex “degree of separation”, is the greatest distance present beetween the vertex and the others. It is computed by: $e_v = max_i \delta^{\rightarrow}(v,i)\delta^{\rightarrow}(i,v)$, with $i \in [1, N-1]$ and thus considering directed shortest path arriving and starting there.

The **in-eccentricity** value cosiders just the greatest shortest path arriving at the vertex, $e_v^{in} = max_i \delta^{\rightarrow}(i,v)$, while the **out-eccentricity** is the opposite, $e_v^{out} = max_i \delta^{\rightarrow}(v,i)$.

3.2 Examples

Statistics of the Connected Network of Figure 1

```
--- Node: V1
Vertex degree abs: 4.0
Indegree abs: 4
Outdegree abs: 4
Degree rel : 1.0
Indegree rel: 1.0
Outdegree rel: 1.0
Weighted degree: 4.0
Weighted indegree: 4.0
Weighted outdegree: 4.0
Degree 2 abs: 4.0
Indegree 2 abs: 4
Outdegree 2 abs: 4
Global centrality: 1.0
Global incentrality: 1.0
Global outcentrality: 1.0
Global weighted centrality: 4.0
Global weighted incentrality: 4.0
Global weighted outcentrality: 4.0
Global centrality corrected: 0.25
Global incentrality corrected: 0.25
Global outcentrality corrected: 0.25
Global centrality weighted & corrected: 16.0
Global incentrality weighted & corrected: 16.0
Global outcentrality weighted & corrected: 16.0
Betweenness: 0.0
Eccentricity: 1
In-eccentricity: 1
Out-eccentricity: 1

--- Node: V2
Vertex degree abs: 4.0
Indegree abs: 4
Outdegree abs: 4
Degree rel : 1.0
```

```
Indegree rel: 1.0
Outdegree rel: 1.0
Weighted degree: 4.0
Weighted indegree: 4.0
Weighted outdegree: 4.0
Degree 2 abs: 4.0
Indegree 2 abs: 4
Outdegree 2 abs: 4
Global centrality: 1.0
Global incentrality: 1.0
Global outcentrality: 1.0
Global weighted centrality: 4.0
Global weighted incentrality: 4.0
Global weighted outcentrality: 4.0
Global centrality corrected: 0.25
Global incentrality corrected: 0.25
Global outcentrality corrected: 0.25
Global centrality weighted & corrected: 16.0
Global incentrality weighted & corrected: 16.0
Global outcentrality weighted & corrected: 16.0
Betweenness: 0.0
Eccentricity: 1
In-eccentricity: 1
Out-eccentricity: 1

--- Node: V3
Vertex degree abs: 4.0
Indegree abs: 4
Outdegree abs: 4
Degree rel : 1.0
Indegree rel: 1.0
Outdegree rel: 1.0
Weighted degree: 4.0
Weighted indegree: 4.0
Weighted outdegree: 4.0
Degree 2 abs: 4.0
Indegree 2 abs: 4
Outdegree 2 abs: 4
Global centrality: 1.0
Global incentrality: 1.0
Global outcentrality: 1.0
Global weighted centrality: 4.0
Global weighted incentrality: 4.0
Global weighted outcentrality: 4.0
Global centrality corrected: 0.25
Global incentrality corrected: 0.25
Global outcentrality corrected: 0.25
Global centrality weighted & corrected: 16.0
Global incentrality weighted & corrected: 16.0
Global outcentrality weighted & corrected: 16.0
Betweenness: 0.0
Eccentricity: 1
In-eccentricity: 1
Out-eccentricity: 1

--- Node: V4
Vertex degree abs: 4.0
Indegree abs: 4
Outdegree abs: 4
Degree rel : 1.0
Indegree rel: 1.0
Outdegree rel: 1.0
```

```
Weighted degree: 4.0
Weighted indegree: 4.0
Weighted outdegree: 4.0
Degree 2 abs: 4.0
Indegree 2 abs: 4
Outdegree 2 abs: 4
Global centrality: 1.0
Global incentrality: 1.0
Global outcentrality: 1.0
Global weighted centrality: 4.0
Global weighted incentrality: 4.0
Global weighted outcentrality: 4.0
Global centrality corrected: 0.25
Global incentrality corrected: 0.25
Global outcentrality corrected: 0.25
Global centrality weighted & corrected: 16.0
Global incentrality weighted & corrected: 16.0
Global outcentrality weighted & corrected: 16.0
Betweenness: 0.0
Eccentricity: 1
In-eccentricity: 1
Out-eccentricity: 1

--- Node: V5
Vertex degree abs: 4.0
Indegree abs: 4
Outdegree abs: 4
Degree rel : 1.0
Indegree rel: 1.0
Outdegree rel: 1.0
Weighted degree: 4.0
Weighted indegree: 4.0
Weighted outdegree: 4.0
Degree 2 abs: 4.0
Indegree 2 abs: 4
Outdegree 2 abs: 4
Global centrality: 1.0
Global incentrality: 1.0
Global outcentrality: 1.0
Global weighted centrality: 4.0
Global weighted incentrality: 4.0
Global weighted outcentrality: 4.0
Global centrality corrected: 0.25
Global incentrality corrected: 0.25
Global outcentrality corrected: 0.25
Global centrality weighted & corrected: 16.0
Global incentrality weighted & corrected: 16.0
Global outcentrality weighted & corrected: 16.0
Betweenness: 0.0
Eccentricity: 1
In-eccentricity: 1
Out-eccentricity: 1
```

Statistics of the Star Network of Figure 2

```
--- Node: V1
Vertex degree abs: 4.0
Indegree abs: 4
Outdegree abs: 4
Degree rel : 1.0
Indegree rel: 1.0
```

```
Outdegree rel: 1.0
Weighted degree: 4.0
Weighted indegree: 4.0
Weighted outdegree: 4.0
Degree 2 abs: 4.0
Indegree 2 abs: 4
Outdegree 2 abs: 4
Global centrality: 1.0
Global incentrality: 1.0
Global outcentrality: 1.0
Global weighted centrality: 4.0
Global weighted incentrality: 4.0
Global weighted outcentrality: 4.0
Global centrality corrected: 1.0
Global incentrality corrected: 1.0
Global outcentrality corrected: 1.0
Global centrality weighted & corrected: 4.0
Global incentrality weighted & corrected: 4.0
Global outcentrality weighted & corrected: 4.0
Betweenness: 1.0
Eccentricity: 1
In-eccentricity: 1
Out-eccentricity: 1

--- Node: V2
Vertex degree abs: 1.0
Indegree abs: 1
Outdegree abs: 1
Degree rel : 0.25
Indegree rel: 0.25
Outdegree rel: 0.25
Weighted degree: 1.0
Weighted indegree: 1.0
Weighted outdegree: 1.0
Degree 2 abs: 4.0
Indegree 2 abs: 4
Outdegree 2 abs: 4
Global centrality: 0.5714285714285714
Global incentrality: 0.5714285714285714
Global outcentrality: 0.5714285714285714
Global weighted centrality: 7.0
Global weighted incentrality: 7.0
Global weighted outcentrality: 7.0
Global centrality corrected: 0.7272727272727273
Global incentrality corrected: 0.7272727272727273
Global outcentrality corrected: 0.7272727272727273
Global centrality weighted & corrected: 5.5
Global incentrality weighted & corrected: 5.5
Global outcentrality weighted & corrected: 5.5
Betweenness: 0.0
Eccentricity: 2
In-eccentricity: 2
Out-eccentricity: 2

--- Node: V3
Vertex degree abs: 1.0
Indegree abs: 1
Outdegree abs: 1
Degree rel : 0.25
Indegree rel: 0.25
Outdegree rel: 0.25
Weighted degree: 1.0
```

```
Weighted indegree: 1.0
Weighted outdegree: 1.0
Degree 2 abs: 4.0
Indegree 2 abs: 4
Outdegree 2 abs: 4
Global centrality: 0.5714285714285714
Global incentrality: 0.5714285714285714
Global outcentrality: 0.5714285714285714
Global weighted centrality: 7.0
Global weighted incentrality: 7.0
Global weighted outcentrality: 7.0
Global centrality corrected: 0.7272727272727273
Global incentrality corrected: 0.7272727272727273
Global outcentrality corrected: 0.7272727272727273
Global centrality weighted & corrected: 5.5
Global incentrality weighted & corrected: 5.5
Global outcentrality weighted & corrected: 5.5
Betweenness: 0.0
Eccentricity: 2
In-eccentricity: 2
Out-eccentricity: 2

--- Node: V4
Vertex degree abs: 1.0
Indegree abs: 1
Outdegree abs: 1
Degree rel : 0.25
Indegree rel: 0.25
Outdegree rel: 0.25
Weighted degree: 1.0
Weighted indegree: 1.0
Weighted outdegree: 1.0
Degree 2 abs: 4.0
Indegree 2 abs: 4
Outdegree 2 abs: 4
Global centrality: 0.5714285714285714
Global incentrality: 0.5714285714285714
Global outcentrality: 0.5714285714285714
Global weighted centrality: 7.0
Global weighted incentrality: 7.0
Global weighted outcentrality: 7.0
Global centrality corrected: 0.7272727272727273
Global incentrality corrected: 0.7272727272727273
Global outcentrality corrected: 0.7272727272727273
Global centrality weighted & corrected: 5.5
Global incentrality weighted & corrected: 5.5
Global outcentrality weighted & corrected: 5.5
Betweenness: 0.0
Eccentricity: 2
In-eccentricity: 2
Out-eccentricity: 2

--- Node: V5
Vertex degree abs: 1.0
Indegree abs: 1
Outdegree abs: 1
Degree rel : 0.25
Indegree rel: 0.25
Outdegree rel: 0.25
Weighted degree: 1.0
Weighted indegree: 1.0
Weighted outdegree: 1.0
```

```
Degree 2 abs: 4.0
Indegree 2 abs: 4
Outdegree 2 abs: 4
Global centrality: 0.5714285714285714
Global incentrality: 0.5714285714285714
Global outcentrality: 0.5714285714285714
Global weighted centrality: 7.0
Global weighted incentrality: 7.0
Global weighted outcentrality: 7.0
Global centrality corrected: 0.7272727272727273
Global incentrality corrected: 0.7272727272727273
Global outcentrality corrected: 0.7272727272727273
Global centrality weighted & corrected: 5.5
Global incentrality weighted & corrected: 5.5
Global outcentrality weighted & corrected: 5.5
Betweenness: 0.0
Eccentricity: 2
In-eccentricity: 2
Out-eccentricity: 2
```

Statistics of the Star Network of Figure 3

```
--- Node: V1
Vertex degree abs: 3.0
Indegree abs: 3
Outdegree abs: 3
Degree rel : 0.75
Indegree rel: 0.75
Outdegree rel: 0.75
Weighted degree: 3.0
Weighted indegree: 3.0
Weighted outdegree: 3.0
Degree 2 abs: 3.
Indegree 2 abs: 3
Outdegree 2 abs: 3
Global centrality: 1.3333333333333333
Global incentrality: 1.3333333333333333
Global outcentrality: 1.3333333333333333
Global weighted centrality: 3.0
Global weighted incentrality: 3.0
Global weighted outcentrality: 3.0
Global centrality corrected: 1.3333333333333333
Global incentrality corrected: 1.3333333333333333
Global outcentrality corrected: 1.3333333333333333
Global centrality weighted & corrected: 3.0
Global incentrality weighted & corrected: 3.0
Global outcentrality weighted & corrected: 3.0
Betweenness: 0.5
Eccentricity: 1
In-eccentricity: 1
Out-eccentricity: 1

--- Node: V2
Vertex degree abs: 1.0
Indegree abs: 1
Outdegree abs: 1
Degree rel : 0.25
Indegree rel: 0.25
Outdegree rel: 0.25
Weighted degree: 1.0
Weighted indegree: 1.0
```

```
Weighted outdegree: 1.0
Degree 2 abs: 3.0
Indegree 2 abs: 3
Outdegree 2 abs: 3
Global centrality: 0.8
Global incentrality: 0.8
Global outcentrality: 0.8
Global weighted centrality: 5.0
Global weighted incentrality: 5.0
Global weighted outcentrality: 5.0
Global centrality corrected: 1.0
Global incentrality corrected: 1.0
Global outcentrality corrected: 1.0
Global centrality weighted & corrected: 4.0
Global incentrality weighted & corrected: 4.0
Global outcentrality weighted & corrected: 4.0
Betweenness: 0.0
Eccentricity: 2
In-eccentricity: 2
Out-eccentricity: 2

--- Node: V3
Vertex degree abs: 1.0
Indegree abs: 1
Outdegree abs: 1
Degree rel : 0.25
Indegree rel: 0.25
Outdegree rel: 0.25
Weighted degree: 1.0
Weighted indegree: 1.0
Weighted outdegree: 1.0
Degree 2 abs: 3.0
Indegree 2 abs: 3
Outdegree 2 abs: 3
Global centrality: 0.8
Global incentrality: 0.8
Global outcentrality: 0.8
Global weighted centrality: 5.0
Global weighted incentrality: 5.0
Global weighted outcentrality: 5.0
Global centrality corrected: 1.0
Global incentrality corrected: 1.0
Global outcentrality corrected: 1.0
Global centrality weighted & corrected: 4.0
Global incentrality weighted & corrected: 4.0
Global outcentrality weighted & corrected: 4.0
Betweenness: 0.0
Eccentricity: 2
In-eccentricity: 2
Out-eccentricity: 2

--- Node: V4
Vertex degree abs: 1.0
Indegree abs: 1
Outdegree abs: 1
Degree rel : 0.25
Indegree rel: 0.25
Outdegree rel: 0.25
Weighted degree: 1.0
Weighted indegree: 1.0
Weighted outdegree: 1.0
Degree 2 abs: 3.0
```

```
Indegree 2 abs: 3
Outdegree 2 abs: 3
Global centrality: 0.8
Global incentrality: 0.8
Global outcentrality: 0.8
Global weighted centrality: 5.0
Global weighted incentrality: 5.0
Global weighted outcentrality: 5.0
Global centrality corrected: 1.0
Global incentrality corrected: 1.0
Global outcentrality corrected: 1.0
Global centrality weighted & corrected: 4.0
Global incentrality weighted & corrected: 4.0
Global outcentrality weighted & corrected: 4.0
Betweenness: 0.0
Eccentricity: 2
In-eccentricity: 2
Out-eccentricity: 2

--- Node: V5
Vertex degree abs: 0.0
Indegree abs: 0
Outdegree abs: 0
Degree rel : 0.0
Indegree rel: 0.0
Outdegree rel: 0.0
Weighted degree: 0.0
Weighted indegree: 0.0
Weighted outdegree: 0.0
Degree 2 abs: 0.0
Indegree 2 abs: 0
Outdegree 2 abs: 0
Global centrality: 0.0
Global incentrality: 0.0
Global outcentrality: 0.0
Global weighted centrality: 0.0
Global weighted incentrality: 0.0
Global weighted outcentrality: 0.0
Global centrality corrected: 0.0
Global incentrality corrected: 0.0
Global outcentrality corrected: 0.0
Global centrality weighted & corrected: 0.0
Global incentrality weighted & corrected: 0.0
Global outcentrality weighted & corrected: 0.0
Betweenness: 0.0
Eccentricity: 0
In-eccentricity: 0
Out-eccentricity: 0
```

Statistics of the Connected Network of Figure 1

```
--- Node: V1
Vertex degree abs: 1.0
Indegree abs: 1
Outdegree abs: 1
Degree rel : 0.25
Indegree rel: 0.25
Outdegree rel: 0.25
Weighted degree: 1.0
Weighted indegree: 1.0
Weighted outdegree: 1.0
```

```
Degree 2 abs: 2.0
Indegree 2 abs: 2
Outdegree 2 abs: 2
Global centrality: 0.4
Global incentrality: 0.4
Global outcentrality: 0.4
Global weighted centrality: 10.0
Global weighted incentrality: 10.0
Global weighted outcentrality: 10.0
Global centrality corrected: 1.021276595744681
Global incentrality corrected: 1.021276595744681
Global outcentrality corrected: 1.021276595744681
Global centrality weighted & corrected: 3.9166666666666665
Global incentrality weighted & corrected: 3.9166666666666665
Global outcentrality weighted & corrected: 3.9166666666666665
Betweenness: 0.0
Eccentricity: 4
In-eccentricity: 4
Out-eccentricity: 4

--- Node: V2
Vertex degree abs: 2.0
Indegree abs: 2
Outdegree abs: 2
Degree rel : 0.5
Indegree rel: 0.5
Outdegree rel: 0.5
Weighted degree: 2.0
Weighted indegree: 2.0
Weighted outdegree: 2.0
Degree 2 abs: 3.0
Indegree 2 abs: 3
Outdegree 2 abs: 3
Global centrality: 0.5714285714285714
Global incentrality: 0.5714285714285714
Global outcentrality: 0.5714285714285714
Global weighted centrality: 7.0
Global weighted incentrality: 7.0
Global weighted outcentrality: 7.0
Global centrality corrected: 0.9230769230769231
Global incentrality corrected: 0.9230769230769231
Global outcentrality corrected: 0.9230769230769231
Global centrality weighted & corrected: 4.333333333333333
Global incentrality weighted & corrected: 4.333333333333333
Global outcentrality weighted & corrected: 4.333333333333333
Betweenness: 0.5
Eccentricity: 3
In-eccentricity: 3
Out-eccentricity: 3

--- Node: V3
Vertex degree abs: 2.0
Indegree abs: 2
Outdegree abs: 2
Degree rel : 0.5
Indegree rel: 0.5
Outdegree rel: 0.5
Weighted degree: 2.0
Weighted indegree: 2.0
Weighted outdegree: 2.0
Degree 2 abs: 4.0
Indegree 2 abs: 4
```

```
Outdegree 2 abs: 4
Global centrality: 0.6666666666666666
Global incentrality: 0.6666666666666666
Global outcentrality: 0.6666666666666666
Global weighted centrality: 6.0
Global weighted incentrality: 6.0
Global weighted outcentrality: 6.0
Global centrality corrected: 0.8
Global incentrality corrected: 0.8
Global outcentrality corrected: 0.8
Global centrality weighted & corrected: 5.0
Global incentrality weighted & corrected: 5.0
Global outcentrality weighted & corrected: 5.0
Betweenness: 0.6666666666666666
Eccentricity: 2
In-eccentricity: 2
Out-eccentricity: 2

--- Node: V4
Vertex degree abs: 2.0
Indegree abs: 2
Outdegree abs: 2
Degree rel : 0.5
Indegree rel: 0.5
Outdegree rel: 0.5
Weighted degree: 2.0
Weighted indegree: 2.0
Weighted outdegree: 2.0
Degree 2 abs: 3.0
Indegree 2 abs: 3
Outdegree 2 abs: 3
Global centrality: 0.5714285714285714
Global incentrality: 0.5714285714285714
Global outcentrality: 0.5714285714285714
Global weighted centrality: 7.0
Global weighted incentrality: 7.0
Global weighted outcentrality: 7.0
Global centrality corrected: 0.9230769230769229
Global incentrality corrected: 0.9230769230769229
Global outcentrality corrected: 0.9230769230769229
Global centrality weighted & corrected: 4.333333333333334
Global incentrality weighted & corrected: 4.333333333333334
Global outcentrality weighted & corrected: 4.333333333333334
Betweenness: 0.5
Eccentricity: 3
In-eccentricity: 3
Out-eccentricity: 3

--- Node: V5
Vertex degree abs: 1.0
Indegree abs: 1
Outdegree abs: 1
Degree rel : 0.25
Indegree rel: 0.25
Outdegree rel: 0.25
Weighted degree: 1.0
Weighted indegree: 1.0
Weighted outdegree: 1.0
Degree 2 abs: 2.0
Indegree 2 abs: 2
Outdegree 2 abs: 2
Global centrality: 0.4
```

```
Global incentrality: 0.4
Global outcentrality: 0.4
Global weighted centrality: 10.0
Global weighted incentrality: 10.0
Global weighted outcentrality: 10.0
Global centrality corrected: 1.021276595744681
Global incentrality corrected: 1.021276595744681
Global outcentrality corrected: 1.021276595744681
Global centrality weighted & corrected: 3.9166666666666665
Global incentrality weighted & corrected: 3.9166666666666665
Global outcentrality weighted & corrected: 3.9166666666666665
Betweenness: 0.0
Eccentricity: 4
In-eccentricity: 4
Out-eccentricity: 4
```

References

[JAS] JAS, a Java toolkit for creating agent-based simulations - jaslibrary.sourceforge.net.

[JGraphT] JGraphT - jgrapht.sourceforge.net.

[Scott'91] John Schott, *Social Network Analysis, a Handbook*, Sage Publications, London 1991.

[Wasserman&Faust'94] Stanley Wasserman and Katherine Faust, *Social Network Analysis, Methods and Applications*, Cambridge University Press, Cambridge 1994.